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Meson Properties at Finite Temperature in the Linear Sigma Model

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Abstract The meson masses are investigated at finite temperature in the framework of the linear sigma model with an explicit chiral symmetry breaking term. The imaginary-time thermo-field dynamics and effective potential have been used for the calculation of the meson masses. We found that the behavior of the sigma and pion masses at finite temperature is in agreement with previous works. The critical temperature, the order of the phase transition, and the dependence of the meson fields on the temperature are discussed.

Keywords The linear sigma model · The mean field approximation · Finite temperature field theory

1 Introduction

The study of physics at finite temperature is very interesting from both theoretical and experimental points of view since the study has involves properties such as phase transitions, blackbody radiation, etc. According to the standard big bang model, it is believed that a series of phase transitions happened at the early stages of the evolution of the universe, the QCD phase transition being one of them [1, 2].

The appropriate framework for the study of phase transitions is the thermal field theory or the finite temperature field theory, a combination of quantum field theory and statistical mechanics [1]. Within this framework, the finite temperature effective potential is an important and well-used theoretical tool. The use of such techniques goes back to the 1970's when Kirzhnits and Linde [3, 4] first proposed that symmetries broken at zero temperature could be restored at finite temperatures. There are two main paths to study the chiral phase transition, namely lattice QCD methods and effective field theories. Lattice QCD can tell us many things about the phase transition, but cannot be used to study its dynamics [1]. Hence, there is a need for models which do not suffer this restriction. The linear sigma model satisfies these requirements, exhibiting many of the symmetries observed in QCD. The linear

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sigma model was first introduced in the 1960's as a model for pion–nucleon interactions [5] and has attracted much attention recently, especially in studies involving disoriented chiral condensates [6–8]. The model with a $SU(N)_L \times SU(N)_R$ symmetry and two to four quark flavors has been studied in the Hartree approximation [9–11] within the Cornwall–Jackiw–Tomboulis (CJT) formalism [12], and the large-N approximation of the linear sigma model has been investigated by using the same formalism in [13, 14]. In these works, the authors attempt to investigate the degree chiral phase transition and the critical temperature. Hog et al. [15] calculated the sigma and pion masses in the chiral limit of the linear sigma model. The linear sigma model is extended to include the effect of chemical potential on meson masses and the phase transition as in [16, 17]

The aim of this paper is to investigate the meson properties in the linear sigma model with explicit symmetry breaking term at finite temperature. We used the technique in [15] in which the linear sigma model is studied in the chiral limit. We add other investigations such as the order of chiral phase transition and critical temperature and the effect of the temperature on the dynamic of fields which are not calculated in [15] and compare the results with other approaches.

This paper is organized as follows: In Sect. 2, the linear sigma model at zero temperature is explained briefly. The linear sigma model at finite temperature is presented in Sect. 3. The numerical calculations and discussion of the results are presented in Sect. 4. The comparison with other works is presented in Sect. 5.

2 Chiral Quark Sigma Model at Zero Temperature

We describe the interactions of quarks with σ- and $\boldsymbol{\pi}$-mesons by Brise and Banerjee [18]. The Lagrangian density is,

$$L(r) = i\overline{\Psi}\partial_\mu \gamma^\mu \Psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi}.\partial^\mu \boldsymbol{\pi}) + g\overline{\Psi}(\sigma + i\gamma_5 \boldsymbol{\tau}.\boldsymbol{\pi})\Psi - U(\sigma, \boldsymbol{\pi}), \quad (1)$$

where

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2)^2 - m_\pi^2 f_\pi \sigma, \quad (2)$$

is the meson-meson interaction potential and Ψ, σ, and $\boldsymbol{\pi}$ are the quark, sigma (scalar, isoscalar) and pion fields (pseudoscalar, isovector), respectively. In the semiclassical or mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we are replacing power and products of the meson fields by corresponding powers and products of their expectation values. The meson-meson interactions in (2) lead to hidden chiral $SU(2) \times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$$\langle \sigma \rangle = f_\pi, \quad (3)$$

where $f_\pi = 92.4$ MeV is the pion decay constant. The final term in (2) is included to break the chiral symmetry. It leads to partial conservation of the axial-vector isospin current (PCAC). The parameters λ^2, ν^2 can be expressed in terms of f_π and the masses of mesons as,

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi^2}, \quad (4)$$

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}. \quad (5)$$

3 Chiral Quark Sigma Model at Finite Temperature

3.1 Meson Masses at Finite Temperature

In this section, we construct the effective potential of the $O(4)$ symmetric linear sigma model using imaginary time thermo-field dynamics as in [15]. In QCD with two-flavor massless quarks, the chiral $SU(2)_L \times SU(2)_R$ symmetry is satisfied in the Lagrangian level. But this symmetry is spontaneously broken to the $SU(2)_V$ symmetry, because the vacuum, i.e., the ground state of the field configuration, violates it. We can see this phenomenon in the $O(4)$ linear sigma model because the $O(4)$ symmetry has the same algebra as $SU(2)_L \times SU(2)_R$.

We first calculate the temperature-dependent effective potential in the one-loop approximation up an order of $o(\frac{m_\sigma^2}{T^2})$, $o(\frac{m^2}{T^2})$ [15]

$$U_{eff}^T = U(\sigma, \boldsymbol{\pi}) + \frac{7\pi^2 T^4}{90} + \frac{m^2 T^2}{6 f_\pi^2}(\sigma^2 + \boldsymbol{\pi}^2) + \left(\frac{m_\sigma^2 - m_\pi^2}{24 f_\pi^2}\right) T^2 \left(\sigma^2 + \boldsymbol{\pi}^2 - \frac{\nu^2}{2}\right), \quad (6)$$

where the first term on the right side is the contribution from the meson at tree level as defined in (2), the second and third terms are from the quark loop, and the final term is from the meson loop. The pion and sigma fields in (6) should be understood as the background fields which satisfy the classical equation of motion. Since the chiral phase transition temperature T_c is larger than m where $m_\sigma^2 = (2m)^2 + m_\pi^2$, when $\nu^2 \to f_\pi^2$ we will obtain in the case of chiral limit ($m_\pi = 0$) the same results in [15]. By introducing the dimensionless quantities $x^2 = \frac{\boldsymbol{\pi}^2}{f_\pi^2}$ and $y^2 = \frac{\sigma^2}{f_\pi^2}$, the effective potential can be rewritten as

$$\begin{aligned} U_{eff}^T &= U^0(y^2, x^2) + \frac{7\pi^2 T^4}{90} + \frac{(mT)^2}{6 f_\pi^2}((y f_\pi)^2 + (x f_\pi)^2) \\ &\quad + \frac{m_\sigma^2 - m_\pi^2}{24 f_\pi^2} T^2 \left((y f_\pi)^2 + (x f_\pi)^2 - \frac{\nu^2}{2}\right), \end{aligned} \quad (7)$$

the finite temperature vacuum can be defined by externalizing the effective potential as

$$\left.\frac{\partial U_{eff}^T}{\partial \sigma}\right|_{\sigma=\sigma_0(T), \boldsymbol{\pi}=0} = 0, \quad (8)$$

or using the dimensionless variables $\frac{\partial U_{eff}^T}{\partial \sigma}|_{\sigma=\sigma_0(T),\boldsymbol{\pi}=0} = \frac{1}{f_\pi}\frac{\partial U_{eff}^T}{\partial y}|_{y=y_0, x=0} = 0$, we can obtain

$$\lambda^2 f_\pi^4 y_0^3 - \lambda^2 \nu^2 f_\pi^2 y_0 + \frac{1}{3} m^2 T^2 y_0 + \frac{1}{12} m_\sigma^2 T^2 y_0 - m_\pi^2 f_\pi^2 = 0. \quad (9)$$

We obtain the sigma and pion masses as the second derivative of the effective potential [2, 15]:

$$\begin{aligned} m_\sigma^2(T) &= \left.\frac{\partial^2 U_{eff}^T}{\partial \sigma^2}\right|_{\sigma=\sigma_0(T),\boldsymbol{\pi}=0} = \left.\frac{1}{f_\pi^2}\frac{\partial^2 U_{eff}^T}{\partial y^2}\right|_{y=y_0, x=0}, \\ m_\sigma^2(T) &= 3\lambda^2 f_\pi^2 y_0^2 - \lambda^2 \nu^2 + \frac{1}{3} m^2 \left(\frac{T}{f_\pi}\right)^2 + \frac{1}{12}(m_\sigma^2 - m_\pi^2)\left(\frac{T}{f_\pi}\right)^2, \end{aligned} \quad (10)$$

$$m_\pi^2(T) = \frac{\partial^2 U_{eff}^T}{\partial \pi^2}\bigg|_{\sigma=\sigma_0(T),\boldsymbol{\pi}=0} = \frac{1}{f_\pi^2}\frac{\partial^2 U_{eff}^T}{\partial x^2}\bigg|_{y=y_0,x=0},$$
$$m_\pi^2(T) = \lambda^2 f_\pi^2 y_0^2 - \lambda^2 \nu^2 + \frac{1}{3}m^2\left(\frac{T}{f_\pi}\right)^2 + \frac{1}{12}(m_\sigma^2 - m_\pi^2)\left(\frac{T}{f_\pi}\right)^2, \tag{11}$$

in the chiral limit ($m_\pi = 0$), we obtain

$$m_\sigma^2(T) = \frac{1}{12}m_\sigma^2(18y_0^2 + 4r^2t^2 + t^2 - 6), \tag{12}$$

$$m_\pi^2(T) = \frac{1}{12}m_\sigma^2(6y_0^2 + 4r^2t^2 + t^2 - 6). \tag{13}$$

Equations (12), (13) are compatible with the original work [15] where $r = \frac{m}{m_\sigma}$ and $t = \frac{T}{f_\pi}$. For the definition of y_0 and $\sigma_0(T)$ for low temperature, we can define it as in [15]

$$\sigma_0(T) = f_\pi(1 + \delta(T)), \qquad y_0 = 1 + \delta(T). \tag{14}$$

By substituting (14) into (9), we obtain

$$\delta(T) = \frac{\lambda^2\nu^2 f_\pi^2 - \frac{1}{3}m^2T^2 - \frac{1}{12}(m_\sigma^2 - m_\pi^2)T^2 - \lambda^2 f_\pi^4 + m_\pi^2 f_\pi^2}{2\lambda^2 f_\pi^4 + m_\pi^2 f_\pi^2}. \tag{15}$$

3.2 The Meson Fields in the Effective Potential

The Lagrangian density with effective potential is given by

$$L(r) = i\overline{\Psi}\partial_\mu\gamma^\mu\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}.\partial^\mu\boldsymbol{\pi}) + g\overline{\Psi}(\sigma + i\gamma_5\boldsymbol{\tau}.\boldsymbol{\pi})\Psi - U_{eff}^T(\sigma,\boldsymbol{\pi}), \tag{16}$$

the sigma field is expanded around the ground state f_π

$$\sigma = \sigma' - f_\pi, \tag{17}$$

we substitute by (17) into (16) we obtain:

$$L(r) = i\overline{\Psi}\partial_\mu\gamma^\mu\Psi + \frac{1}{2}(\partial_\mu\sigma'\partial^\mu\sigma' + \partial_\mu\boldsymbol{\pi}.\partial^\mu\boldsymbol{\pi}) - g\overline{\Psi}f_\pi\Psi + g\overline{\Psi}\sigma'\Psi + ig\overline{\Psi}\gamma_5\boldsymbol{\tau}.\boldsymbol{\pi}\Psi$$
$$- U_{eff}^T(\sigma',\boldsymbol{\pi}), \tag{18}$$

with

$$U_{eff}^T = U(\sigma',\boldsymbol{\pi}) + \frac{7\pi^2T^4}{90} + \frac{m^2T^2}{6f_\pi^2}\big((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2\big)$$
$$+ \left(\frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2}\right)T^2\left((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2 - \frac{\nu^2}{2}\right). \tag{19}$$

The time-independent fields $\sigma'(r)$ and $\boldsymbol{\pi}(r)$ are to satisfy the Euler-Lagrange equations, and the quark wave function satisfies the Dirac equation. Substituting (18) in the Euler-

Lagrange equation, we get:

$$\Box\sigma' = g\overline{\Psi}\Psi - \lambda^2((\sigma' - f)^2 + \boldsymbol{\pi}^2 - \nu^2)(\sigma' - f)$$
$$- m_\pi^2 f_\pi - \left(\frac{m_\sigma^2 - m_\pi^2}{6f_\pi^2}\right)T^2(\sigma' - f_\pi), \tag{20}$$

$$\Box\boldsymbol{\pi} = ig\overline{\Psi}\gamma_5\boldsymbol{\tau}\Psi - \lambda^2((\sigma' - f)^2 + \boldsymbol{\pi}^2 - \nu^2)\boldsymbol{\pi} - \left(\frac{m_\sigma^2 - m_\pi^2}{6f_\pi^2}\right)T^2\boldsymbol{\pi}, \tag{21}$$

where $\boldsymbol{\tau}$ refers to Pauli isospin-matrices, $\gamma_5 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$.

We used the hedgehog ansatz [19] where

$$\boldsymbol{\pi}(r) = \overset{\leftrightarrow}{\mathbf{r}}\,\pi(r). \tag{22}$$

The Dirac equation for quarks is [19]

$$\frac{du}{dr} = -p(r)u + (W - m_q + S(r))w, \tag{23}$$

where $S(r) = g\langle\sigma'\rangle$, $P(r) = \langle\boldsymbol{\pi}.\hat{\mathbf{r}}\rangle$ and W are the scalar potential, the pseudoscalar potential, and the eigenvalue of the quark spinor Ψ, respectively.

$$\frac{dw}{dr} = -(W - m_q + S(r))u + \left(\frac{2}{r} - p(r)\right)w. \tag{24}$$

Including the color degrees of freedom, one has $g\overline{\Psi}\Psi \to N_c g\overline{\Psi}\Psi$ where $N_c = 3$ is the color number and g is the coupling constant. The Dirac wave functions $\Psi(r)$ and $\overline{\Psi}(r)$ are given by

$$\Psi(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ iw(r) \end{bmatrix} \quad \text{and} \quad \overline{\Psi}(r) = \frac{1}{\sqrt{4\pi}}[u(r) \quad iw(r)] \tag{25}$$

and the sigma, pion, and vector densities are given by

$$\rho_s = N_c g\overline{\Psi}\Psi = \frac{3g}{4\pi}(u^2 - w^2), \tag{26}$$

$$\rho_p = iN_c g\overline{\Psi}\gamma_5\vec{\tau}\Psi = \frac{3}{4\pi}g(-2uw), \tag{27}$$

$$\rho_v = \frac{3g}{4\pi}(u^2 + w^2). \tag{28}$$

The boundary conditions for the asymptote at $\sigma(r)$ and $\pi(r)$ at $r \to \infty$ are:

$$\sigma(r) \sim -f_\pi, \qquad \pi(r) \sim 0. \tag{29}$$

4 Numerical Calculations

4.1 The Scalar Field σ'

To solve (20), we integrate a suitable Green's function over the source fields as in [19–21], thus

$$\sigma'(\mathbf{r}) = \int d^3\mathbf{r}' D_\sigma(\mathbf{r} - \grave{\mathbf{r}})\bigg(g\rho_s(\grave{\mathbf{r}}) - \lambda^2((\sigma' - f)^2 + \boldsymbol{\pi}^2 - \nu^2)(\sigma' - f) - m_\pi^2 f_\pi - \left(\frac{m_\sigma^2 - m_\pi^2}{6f_\pi^2}\right)T^2(\sigma' - f_\pi)\bigg) \quad (30)$$

where

$$D_\sigma(\mathbf{r} - \grave{\mathbf{r}}) = \frac{1}{4\pi|\mathbf{r} - \grave{\mathbf{r}}|}\exp(-m_\sigma|\mathbf{r} - \grave{\mathbf{r}}|).$$

The scalar field is spherical in this model as we only need the $l = 0$ term

$$D_\sigma(\mathbf{r} - \grave{\mathbf{r}}) = \frac{1}{4\pi}\sinh(m_\sigma r_<)\frac{\exp(-m_\sigma r_>)}{r_>}, \quad (31)$$

therefore we arrive at the integral equation for $\sigma'(\mathbf{r})$:

$$\sigma'(\mathbf{r}) = m_\sigma \int_0^\infty r'^2 dr' \left(\frac{\sinh(m_\sigma r_>)}{m_\sigma r_>}\frac{\exp(-m_\sigma r_>)}{m_\sigma r_>}\right)\bigg(g\rho_s(\grave{\mathbf{r}}) - \lambda^2((\sigma' - f)^2 + \boldsymbol{\pi}^2 - \nu^2)(\sigma' - f) - m_\pi^2 f_\pi - \left(\frac{m_\sigma^2 - m_\pi^2}{6f_\pi^2}\right)T^2(\sigma' - f_\pi)\bigg). \quad (32)$$

We will solve this equation by iterating to self-consistency.

4.2 The Pion Field $\boldsymbol{\pi}$

To solve (21), we integrate a suitable Green's function over the source fields. We use the $l = 1$ component of the pion Green's function. Thus

$$\boldsymbol{\pi}(r) = m_\pi \int_0^\infty r'^2 dr' \frac{[-\sinh(m_\pi r_<) + m_\pi r_< \cosh(m_\pi r_<)]}{(m_\pi r_>)^2}\bigg(g\rho_p(\grave{r}) - \lambda^2((\sigma' - f)^2 + \boldsymbol{\pi}^2 - \nu^2)\boldsymbol{\pi} - \left(\frac{m_\sigma^2 - m_\pi^2}{6f_\pi^2}\right)T^2\boldsymbol{\pi}\bigg). \quad (33)$$

We have solved the Dirac equations (23), (24) using the fourth-order Rung Kutta method. Due to the implicit nonlinearity of (32), (33), it is necessary to iterate the solution until self-consistency is achieved. To start this iteration process, we use the chiral circle form for the meson fields [19]

$$S(r) = m_q(1 - \cos\theta), \qquad P(r) = -m_q \sin\theta, \quad (34)$$

where $\theta = \tanh r$ and m_q is the quark mass.

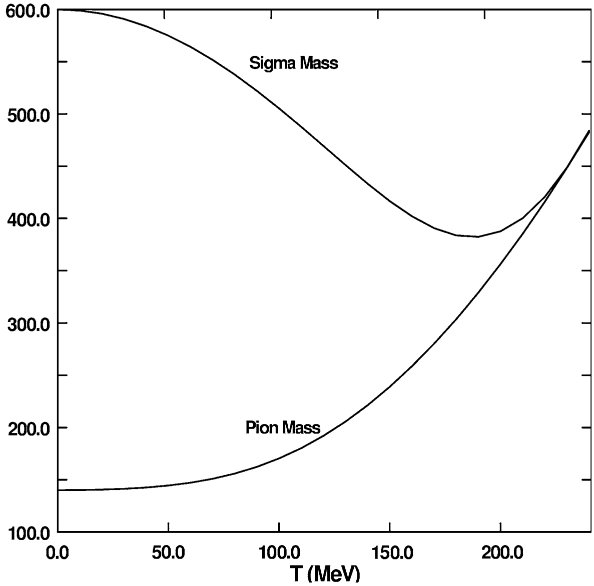

Fig. 1 The solution of the system of gap equations in the case of broken chiral symmetry. At zero temperature, the pions appear with the observed masses

4.3 Discussion of Results

We concentrate only on the thermal effects and ignore the quantum corrections. It is important to examine the linear sigma model in the presence of the symmetry breaking term, which is realistic and reproduces the quark mass.

In Fig. 1, we find that the sigma mass $m_\sigma(T)$ monotonically decreases as a function of temperature T. The observed mass ($m_\sigma = 600$ MeV) at zero temperature is shown in the figure and the pion mass monotonically increases with temperature T. At higher temperatures of more than 226.33 MeV, the sigma and pion masses have the same masses, hence the behavior is in agreement with the previous calculations in [1, 2]. They obtained the massive pion mass ($m_\pi = 140$ MeV) in the presence of a symmetry breaking term at zero temperature.

In Fig. 2, we show the order parameter $\Phi = (\sigma^2 + \boldsymbol{\pi}^2)^{\frac{1}{2}}$ of the chiral phase transition in the chiral limit where it monotonically decreases and is a single-valued function of the temperature T; therefore the order of phase transition is as a second-order phase transition. This situation is similar to the behavior of the order parameter in the large N in the linear sigma model as in [1].

In Figs. 3, 4, we examine the dependence of the sigma and pion fields on temperature T, which it has not been examined in previous calculations as in [1, 2]. In our work, we used the same iteration method to calculate the pion and sigma fields in the case of the effective potential as we used before at zero temperature as in [19–21]. In Fig. 3, the sigma field is plotted as a function of distance r for different values of temperature $T = (0, 150, 250)$ MeV. The sigma field at the finite temperature takes the same behavior at zero temperature. The increase in the temperature leads to an increase in the values of sigma field, particularly in the range of $r = (0.5\text{–}3)$ fm. In Fig. 4, the pion field represented by a P-wave function for all the different values of temperature and temperature effects on the value of the pion field are only present when the increase in the temperature leads to an increase in the value of pion field. Furthermore, we study the effect of temperature on the effective potential. From

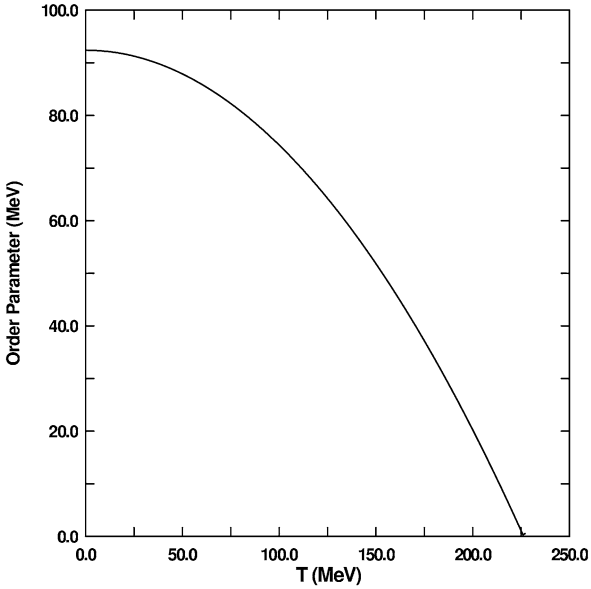

Fig. 2 The behavior of the order parameter as a function of temperature in the chiral limit

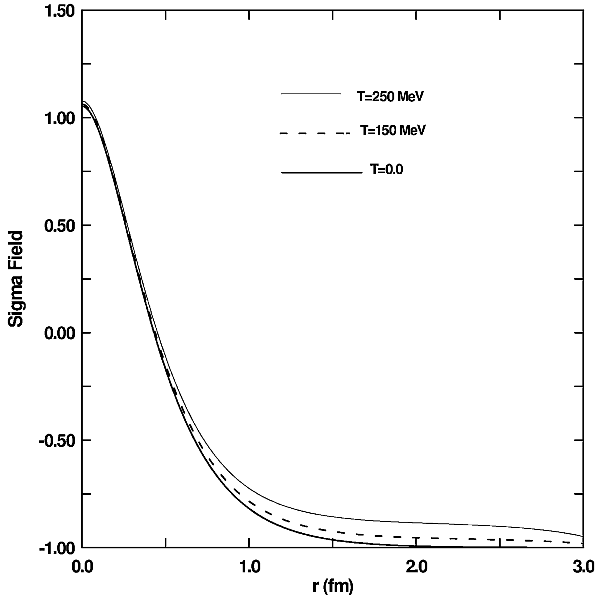

Fig. 3 The behavior of the sigma field as function of distance r at different value of temperature in the case when chiral symmetry is broken

Fig. 5, we see the effective potential is a linear function of distance r and that the effective potential increases by increasing the temperature T at a fixed of value r. This behavior is in agreement with the work of Nemoto et al. [2] where they examined the effect of temperature on the effective potential and deduced that the effective potential increased by increasing the temperature T.

In Fig. 6, the order-parameter phase transition is plotted as a function of distance r for different values of $T = (0, 150, 250)$ MeV. We note that all curves show the same behav-

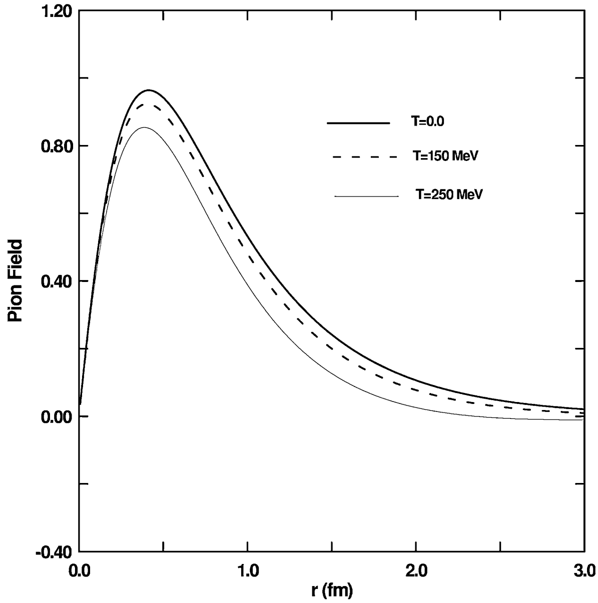

Fig. 4 The behavior of the pion field as function of distance r at different value of temperature in the case when chiral symmetry is broken

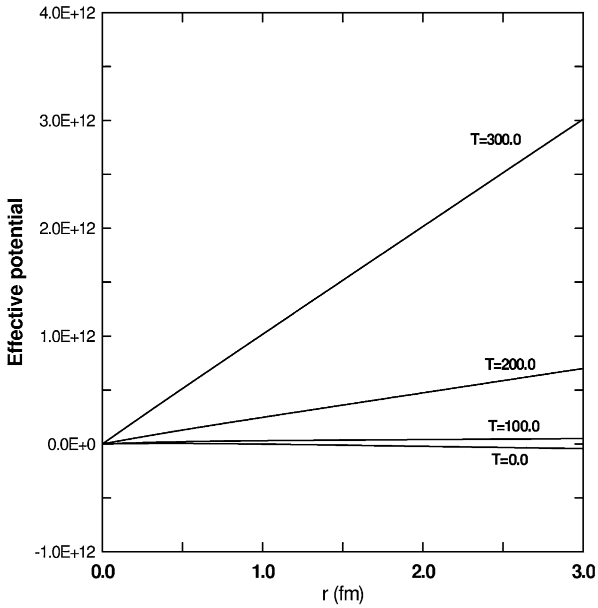

Fig. 5 The behavior of the effective potential as a function of distance r at different values of temperature in the case when chiral symmetry is broken

ior from zero temperature to finite temperature where the order-parameter decreases in the range ($r = 0 \rightarrow 0.7$) fm and increases in the range ($r = 0.7 \rightarrow 3.0$) fm. We note that the temperature strongly affects the values of the order parameter where an increase in the value of the temperature leads to a decrease in the value of the order-parameter. In this work, we calculate the critical temperature T_c. From [1, 2], we can define the critical temperature as the value of the temperature where the sigma and pion masses have the same effective mass.

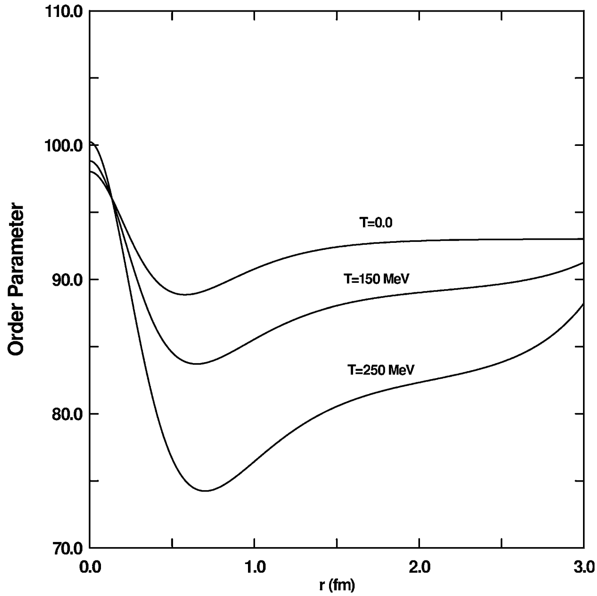

Fig. 6 The behavior of the order parameter as function of distance r at different value of temperature in the case when chiral symmetry is broken

By solving (10), (11) numerically, we can obtain the value $T_c = 226.33$ MeV as seen in Fig. 2 where the order-parameter crosseds the temperature axis at 226.33 MeV.

5 Comparison with Other Works

The finite temperature in the linear sigma is widely studied in many works such as [1–3] so it is important to compare the present results with other approaches.

Petropoulos [1] found the phase is a first-order phase transition in the chiral limit ($m_\pi = 0$), but the limit for large N is taken in the CJT approach, the result is a second-order phase transition; therefore the last result is in agreement with the present result at the large N limit. He calculated a critical temperature equal to 131.5 MeV. We obtained a critical temperature $T_c = 226.33$ MeV. The difference in the two values depends on the loops in the effective potential. In addition, the lattice QCD calculation shows that chiral symmetry is restored at approximately $T = 100$–300 MeV [2].

Nemoto et al. [2] used the Cornwall-Jackiw-Tomboulis (CJT) effective potential where they defined the meson mass as the second derivative of the effective potential which as done in the present work. The critical temperature was calculated to be $T_c = 230$ MeV, which agrees with our result of $T_c = 226.33$ MeV. The first-order transition takes place in the $O(4)$, but if we take the large N limit in the CJT approach, the result is the second-order. In [22–24], the phase transition was predicted to be a second order phase transition based on the argument that the linear sigma model belongs in the same universality class, which agrees with our result. Arnold and Espina [25] stated that the order of phase transition depends on the loops which represent the effective potential, hence the order of phase transition can not be trusted. In comparison with Hong et al. [15] where they calculated the sigma and pion masses in the chiral limit only, we added more investigations in the presence of the symmetry breaking term in the $O(4)$ and examined the order of the phase transition. Furthermore, we examined the effect of temperature on the pion and sigma fields and the effective potential which was not calculated in [15].

In the work of Chiku and Hastsuda [26], optimal perturbation theory is employed in order to perform the calculations. They obtained gap equations for the effective masses which go further than our Hartree approximation. They included quantum fluctuations as well. So we can not compare our results in full. However, they did not calculate the effective potential. In the work of Caldas et at. [27], a first-order phase transition in the chiral limit was shown.

In the previous calculations as in [1–3], the effect of temperature on the pion and sigma fields was not calculated and hence we investigated these points in the present work.

6 Summary and Conclusion

The chiral phase transition and sigma and pion masses are examined in the framework of linear sigma model in the presence of a symmetry breaking term. From the results, we summarize the following points

- The critical temperature is calculated to be equal to 226.33 MeV, which agrees with previous calculations as discussed in the section above.
- The order-parameter phase transition is investigated which is second-order in the chiral limit.
- The pion and sigma fields and the effective potential are examined respect to temperature. These are not calculated in previous calculations as in [1, 2, 16].
- The gap equations are constructed directly from the effective potential as in the original work [15] and the accuracy in the results are assured.
- We extended the iteration method which is used at zero temperature to describe the behavior of the fields at finite temperatures.

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References

1. Petropoulos, N.: PhD thesis submitted to Manchester University in Oct 2000. hep-ph/0402136 and references therein
2. Nemoto, Y., Naito, K., Oka, M.: Eur. Phys. J. A **9**, 245 (2000)
3. Kirzhnits, D.A.: JETP Lett. **15**, 529 (1972)
4. Kirzhnits, D.A., Linde, A.D.: Phys. Lett. B **42**, 471 (1972)
5. Gell-Mann, M., Levy, M.: Nuovo Cimento **16**, 705 (1960)
6. Rajagopal, K., Wilczek, F.: Nucl. Phys. B **399**, 395 (1993)
7. Rajagopal, K., Wilczek, F.: Nucl. Phys. B **404**, 577 (1993)
8. Kuraev, E.A., Silagadze, Z.K.: Acta Phys. Pol. B **34**, 4019 (2003)
9. Petropoulos, N.: J. Phys. G **25**, 2225 (1999)
10. Lenaghan, J.T., Rischke, D.H., Schaffner-Bielich, J.: Phys. Rev. D **62**, 085008 (2000)
11. Roder, D., Ruppert, J., Rischke, D.H.: Phys. Rev. D **68**, 016003 (2003)
12. Amelino-Camelia, G., Pi, S.Y.: Phys. Rev. D **47**, 2356 (1993)
13. Amelino-Camelia, G.: Phys. Lett. B **407**, 268 (1997)
14. Lenaghan, J.T., Rischke, D.H.: J. Phys. G **26**, 431 (2000)
15. Hong, C., Bo, L., Jiang, H.-q.: Chin. Phys. Lett. **14**, 645 (1997)
16. Mao, H., Petropoulos, N., Shu, S., Zhao, W.: hep-ph/0606241
17. Scavenius, O., Mocsy, A., Mishustin, I.N., Rischke, D.H.: Phys. Rev. C **64**, 045202 (2001)
18. Birse, M., Banerjee, M.: Phys. Rev. D **31**, 118 (1985)
19. Rashdan, M., Abu-Shady, M., Ali, T.S.T.: Inter. J. Mod. Phys. A **22**, 2673 (2007)
20. Abu-Shady, M.: Mod. Phys. Lett. A **24**, 20 (2009)
21. Abu-Shady, M.: Int. J. Theor. Phys. **48**, 1110 (2009)

22. Rajagopal, K., Wilczek, F.: Nucl. Phys. B **399**, 395 (1993)
23. Nakkagawa, H., Yokota, H.: Mod. Phys. Lett. A **11**, 2259 (1996)
24. Berges, J., Jungnickel, D., Wetterich, C.: Phys. Rev. D **59**, 034010 (1999)
25. Arnold, P., Espinosa, O.: Phys. Rev. D **47**, 3546 (1993)
26. Chiku, S., Hatsuda, T.: Phys. Rev. D **57**, 6 (1998)
27. Caldas, H.C., Mota, A.L., Nemes, M.C.: Phys. Rev. D **63**, 056011 (2001)